MATH 8
SAMPLE TEST UNIT 4
(6.4, 6.6, CHP 7)

100 POINTS

NAME: ____________________________

Show your work on this paper. EXACT answers are expected unless otherwise specified. No Graphing Calculators. No scratch paper

Fill in the blanks. (2 points each)

(1) Give an identity for $\cos(2\theta) =$ $\begin{cases} 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \\ \cos^2\theta - \sin^2\theta \end{cases}$ any one of these.

(2) Give an identity for $\sin(\alpha - \beta) =$ $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

(3) Give an identity for $\cos\left(\frac{\theta}{2}\right) =$ $\pm\sqrt{\frac{1+\cos\theta}{2}}$

using $\sin 2\theta = 2\sin\theta\cos\theta$

(4) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12} =$ $\sin\left(2\cdot\frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$ (exact, simplify)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(5) $\cos 12^\circ \cos 18^\circ - \sin 12^\circ \sin 18^\circ =$ $\cos(12^\circ + 18^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$. (exact, simplify)

(6) True or False: $\frac{\sin(4\theta)}{4}$ simplifies to $\sin\theta$ False

(7) Express $\sin 3\theta \cos 7\theta$ as a sum $\frac{1}{2}(\sin 10\theta + \sin(-4\theta))$ or $\frac{1}{2}(\sin 10\theta - \sin 4\theta)$

$\frac{1}{2}(\sin 10\theta + \sin(-4\theta))$ product to sum formula

(8) Using identities, find the exact, simplified value of: (2 points each)
(You must show work, for credit. Calculators should not be used on this problem)

(a) $\tan\left(\frac{-\pi}{12}\right)$ $\frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\frac{3\pi}{12} - \frac{4\pi}{12} = -\frac{\pi}{12}$

$\frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

$\tan\left(\frac{-\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{3}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{3}}$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}}$

(b) $\cos 157.5^\circ$ ________

$\cos(157.5^\circ) = \cos\left(\frac{315}{2}\right)^\circ$

$= \pm\sqrt{\frac{1+\cos 315^\circ}{2}}$

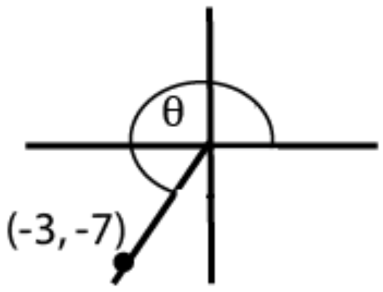
$= -\sqrt{\frac{1 + -\frac{\sqrt{2}}{2}}{2}}$

$= -\frac{\sqrt{2-\sqrt{2}}}{2}$

(9) Simplify: $\frac{\tan\theta + \cot\theta}{3\sec\theta\csc\theta}$ (simplifies to a number) (4 points)

$\frac{\tan\theta + \cot\theta}{3\sec\theta\csc\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{3\cdot\frac{1}{\cos\theta}\frac{1}{\sin\theta}} \quad \frac{\cos\theta\sin\theta}{\cos\theta\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{3} = \frac{1}{3}$

(10) Given the following information about θ, (6 points)



$r=\sqrt{(-3)^2+(-7)^2}=\sqrt{58}$

$\cos\theta = \frac{-3}{\sqrt{58}} \quad \sin\theta = \frac{-7}{\sqrt{58}}$

$180^\circ < \theta < 270^\circ$
$90^\circ < \frac{\theta}{2} < 135^\circ$

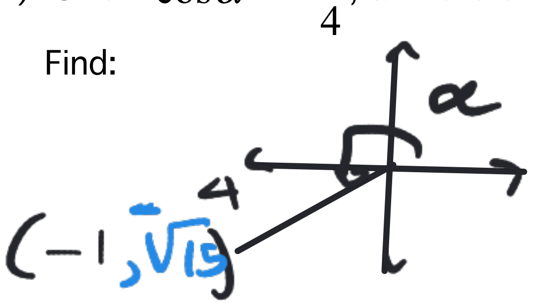
Find a) $\cos(2\theta)$ b) $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

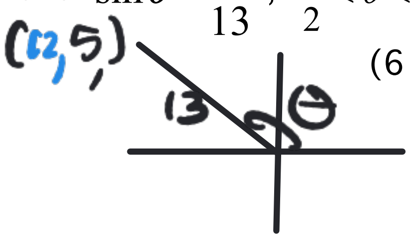
$\cos(2\theta) = 2\cos^2\theta - 1$
$= 2\left(\frac{-3}{\sqrt{58}}\right)^2 - 1 = \frac{18}{58} - 1$
$= \boxed{\frac{-20}{29}}$

$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\frac{-3}{\sqrt{58}}}{2}} = \boxed{\sqrt{\frac{\sqrt{58}-3}{2\sqrt{58}}}}$

(11) Given $\cos\alpha = -\frac{1}{4}$, α in the third quadrant, and $\sin\theta = \frac{5}{13}$, $\frac{\pi}{2} < \theta < \pi$

Find: (6 points)





$\tan\theta = \frac{5}{12}$

a) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= -\frac{1}{4}\cdot\frac{12}{13} + \frac{-\sqrt{15}}{4}\cdot\frac{5}{13} = \boxed{\frac{-12-5\sqrt{15}}{54}}$

b) $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\frac{5}{12}}{1-\frac{25}{144}} \quad \frac{144}{144} = \boxed{\frac{120}{119}}$

(12) Prove the following identity. Presentation should be very clear. (6 points)

$$1-\frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$$

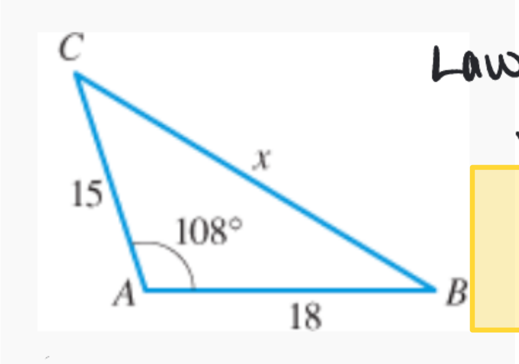
$\cos^2\theta + \sin^2\theta = 1$
$\Rightarrow 1-\sin^2\theta = \cos^2\theta$

$1-\frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta}{1+\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta-\sin^2\theta}{1+\cos\theta} = \frac{1-\sin^2\theta+\cos\theta}{1+\cos\theta}$

$= \frac{\cos^2\theta+\cos\theta}{1+\cos\theta} = \frac{\cos\theta(\cos\theta+1)}{1+\cos\theta} = \cos\theta$

$\therefore$ LHS = RHS

(13) Find x, exactly. Then give the approximate value to 3 decimal places: (4 points)



Law of Cosines

$x^2 = 15^2 + 18^2 - 2(15)(18)\cos 108°$

$$\boxed{\begin{array}{l} x = \sqrt{549 - 540\cos 108°} \\ \approx 26.756 \end{array}}$$

(14). Give the simplified, exact value of the remaining parts for all possible triangles satisfying the given conditions. $\angle A = 30°$, $a = 6$ $c = 6\sqrt{3}$ (No need to put values into your calculator for approximations) (6 points)

$$\frac{\sin A}{a} = \frac{\sin C'}{c}$$

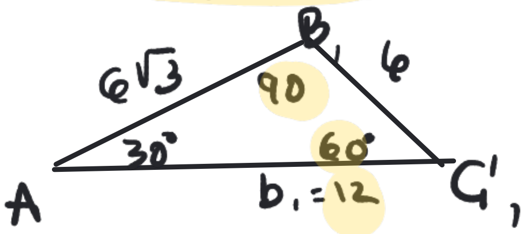
$$\frac{\sin 30°}{6} = \frac{\sin C'}{6\sqrt{3}}$$

$$\frac{6\sqrt{3}\sin 30°}{6} = \sin C'$$

$$\frac{\sqrt{3}}{2} = \sin C'$$

2 triangles

$C_1 = 60°$



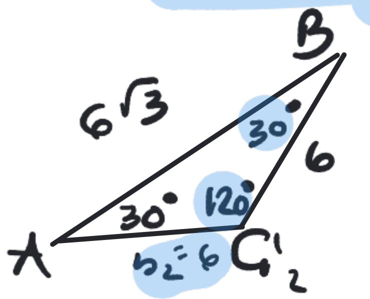
Angle B_1: $180 - 30° - 60° = 90° = B_1$

$$\frac{b_1}{\sin B_1} = \frac{6}{\sin 30°}$$

$$\frac{b_1}{\sin 90°} = \frac{6}{\sin 30°}$$

$b = 12$

$C'_2 = 120°$



$B_2 = 180° - 30° - 120° = 30° = B_2$

$b_2 = 6$

Find all solutions to the following equations. (6 points each)

(15) $\cos(2x) = 2 + 5\cos x$

$2\cos^2 x - 1 = 2 + 5\cos x$

$2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

$2\cos x + 1 = 0 \qquad \cos x - 3 = 0$

$\cos x = -1/2 \qquad \cos x = 3 \to$ no soln

$\boxed{x = \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k}$

(16) $\sin(5x) - \sin(3x) = 0$

sum to product

$2\sin\left(\frac{5x-3x}{2}\right)\cos\left(\frac{5x+3x}{2}\right) = 0$

$2\sin x \cos 5x = 0$

$\sin x = 0 \qquad \cos 5x = 0$

$5x = \frac{\pi}{2} + \pi k$

$\boxed{x = \pi k \qquad x = \frac{\pi}{10} + \frac{\pi k}{5}}$

SOLVE the following equations: $0 \le x < 2\pi$ (6 points each)

$0 \le \theta < 2\pi$

(17) $\sin\theta + 4\sin(2\theta) = 0$

$\sin\theta + 4 \cdot 2\sin\theta\cos\theta = 0$

$\sin\theta(1 + 8\cos\theta) = 0$

$\sin\theta = 0 \qquad 1 + 8\cos\theta = 0$

$\cos\theta = -\frac{1}{8}$

ref $= \cos^{-1}\left(\frac{1}{8}\right)$

$\boxed{x = 0, \pi \quad \pi - \cos^{-1}\frac{1}{8} \quad \pi + \cos^{-1}\frac{1}{8}}$

(18) $9 - 4\sin^2\theta = 12\cos\theta$

$9 - 4(1 - \cos^2\theta) = 12\cos\theta$

$9 - 4 + 4\cos^2\theta = 12\cos\theta$

$4\cos^2\theta - 12\cos\theta + 5 = 0$

$(2\cos\theta - 1)(2\cos\theta - 5) = 0$

$2\cos\theta - 1 = 0 \qquad 2\cos\theta - 5 = 0$

$\cos\theta = 1/2 \qquad \cos\theta = \frac{5}{2}$

no solution

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

(19) $4\cos(2\theta) - 4 = 0$

$4\cos 2\theta = 4$

$\cos 2\theta = 1$

Note: you don't need to apply the identity here.

$2\theta = 0, 2\pi, 4\pi, 6\pi, \ldots$

$\boxed{\theta = 0, \pi}, \cancel{2\pi}, \cancel{3\pi}$

(20) $\sec^2 x - 3\tan^2 x = -5$

$\tan^2 x + 1 - 3\tan^2 x = -5$

$-2\tan^2 x = -6$

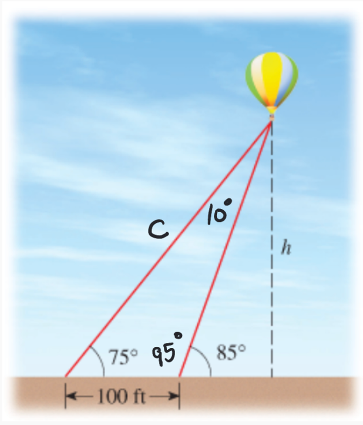
$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(21) (7 points)

Two wires tether a balloon to the ground, as shown. How high is the balloon above the ground?



There are different ways to approach this. First I found angles of red triangle. Then I found c using the red triangle.

$$\frac{c}{\sin 95^\circ} = \frac{100}{\sin 10^\circ}$$

$c = \frac{100}{\sin 10^\circ} \sin 95^\circ$ (no need to put this in calculator)

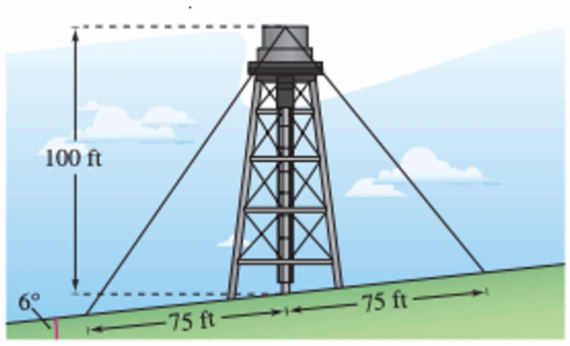
Then looking at big right △

$$\frac{h}{c} = \sin 75^\circ$$

$$\boxed{h = c\sin 75^\circ = \frac{100}{\sin 10^\circ} \sin 95^\circ \sin 75^\circ}$$

(22) (7 points)

Length A 100-foot vertical tower is to be erected on the side of a hill that makes a 6° angle with the horizontal (see figure). Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.



((7 points)

It can be helpful to draw the figure without the extra parts

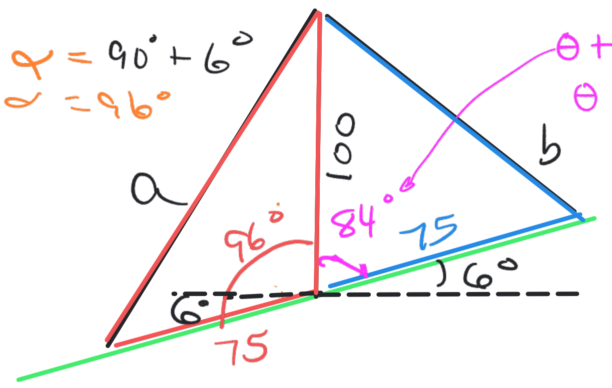
Find angles

$\alpha = 90^\circ + 6^\circ$

$\alpha = 96^\circ$

$\theta + 6^\circ = 90^\circ$

$\theta = 84^\circ$



$a^2 = 75^2 + 100^2 - 2(75)(100)\cos 96^\circ$

$a^2 = 15625 - 15000\cos 96^\circ$

$$\boxed{\begin{aligned} a &= \sqrt{15625 - 15000\cos 96^\circ} \\ a &\approx 131.1 \text{ ft} \end{aligned}}$$

$b^2 = 75^2 + 100^2 - 2 \cdot 75 \cdot 100 \cos 84^\circ$

$$\boxed{\begin{aligned} b &= \sqrt{15625 - 15000\cos 84^\circ} \\ b &\approx 118.6 \text{ ft} \end{aligned}}$$